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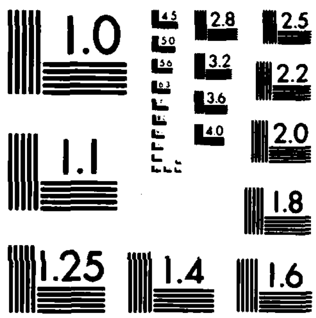

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DIFFUSION APPROXIMATION IN ARROW'S MODEL OF
EXHAUSTABLE RESOURCES

by

Darrell Duffie and Michael Taksar

Technical Report No. 416
August 1983

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY
Contract ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMICS SERIES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
Stanford, California
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DIFFUSION APPROXIMATION IN ARROW'S MODEL OF EXHAUSTABLE RESOURCES*

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1. Introduction

What we shall call "Arrow's Model" for resource exploration and use under uncertainty was first formulated by Arrow [1977] and analyzed by Arrow and Chang [1978]. This model proved to be very successful and stimulated a lot of research. The most interesting further analyses of Arrow's model and some numerical results have since been produced by Hagan, Caflisch, and Keller [1980] (henceforth "H-C-K"), and independently by Derzko and Sethi [1981]. The purpose of this paper is to bring to a "common denominator" the results of previous studies and to introduce a new diffusion model approach to the same problem. The problem is posed roughly as follows.

A central planner charged with the exploration and use of some natural resource (for example, oil) has knowledge of the current stockpile of the resource $(R > 0)$, the remaining unexplored land $(A > 0)$, and the probability distribution of undiscovered resources over the remaining land. For simplicity it is assumed that the random quantities of the resource to be extracted from any two disjoint sections of land of equal area are independently and identically distributed. Then any exploration policy is merely a specification for a decreasing process

*This research was supported by the Office of Naval Research Grant ONR-N00014-79-C-0685 at the Institute for Mathematical Studies in the Social Sciences, Stanford University.

$A = \{A(t);\ t \geq 0\}$, where $A(t)$ is the amount of unexplored land remaining at time t when policy A is followed. We define $N^A(t)$ to be the (random) cumulative amount of resource extractd by time t when the policy A is followed.

The economy has utility at the rate $u(c)$ for consuming the resource at the rate $c \geq 0$, where $u(\cdot)$ is differentiable, increasing, and strictly concave. With time discounting at the rate $\rho > 0$, the total utility for a particular path of consumption $\{c_t, t \geq 0\}$ is then:

$$\int_0^\infty e^{-\rho t} u(c_t)dt \quad .$$

Exploration requires inputs of "goods in general" at a constant ratio of $P > 0$ per unit of land explored. Arrow and Chang argue that we can also scale $u(\cdot)$ to be approximatley linear in terms of goods in general. Under the standard von Neumann-Morgenstern axioms for expected utility maximization we can now formulate the problem as

$$(1.1) \qquad V(A_0,R_0) = \underset{A,c}{\text{Max}}\ E^{A,c}_{A_0,R_0} \int_0^\infty e^{-\rho t}[u(c_t)dt + PdA(t)]$$

subject to:

$$(1.2) \quad R(t) = \int_0^t [-c_s ds + dN^A(s)] \geq 0 \ \forall\, t \geq 0 \quad , \quad R(0) = R_0;\ A(0) = A_0 \quad ,$$

where $E^{A,c}_{x,y}$ denotes expectation under the exploration policy A and the consumption policy c, with initial resource stocks x and initial

unexplored land y. Note that the second (Stieltjes) integral of (1.1) is negative and decreasing, since A is decreasing.

In each of the above cited papers resources are located at discrete points (in "lumps") according to a Poisson distribution. See Feller [1968] page 159 for a good interpretation of Poisson distributions of points in Euclidean space. Thus, for example, the exploration policy $A(t) \equiv A_0 - t$ would generate new discoveries at Poisson arrival times.

Both Arrow and Chang [1978] and Derzko and Sethi [1981] assume that each discovery yields a fixed amount of the resource, while H-C-K allow the size of the discovery to be random according to some general probability distribution with positive support.

One of the purposes of this paper is to reformulate N^A as a controlled diffusion process and obtain the insights which this allows. To do this requires a careful specification of what "instantaneous control" means for Arrow's model. This also serves to place a more rigorous foundation under the previously cited studies. For the remainder of this section, however, we will proceed at a heuristic level to give the basic flavor of stochastic dynamic programming applied to Arrow's model. All of the above mentioned papers proceed roughly along the following lines.

Given the (i.i.d.) probability assumptions, the usual line of attack of Markovian stochastic dynamic programming can be taken. Under an optimal policy, at any time t, $V(A_t,R_t)$ from (1.1) (reinitialized at time t) can be equated with the expected accumulation of utility over

any interval of time Δt, plus the expected discounted value of V at time $t + \Delta t$. There are some formal mathematical difficulties (which are not resolved in any of the previous studies) in taking the limit of this sum divided by Δt as $\Delta t \downarrow 0$. These difficulties are resolved by the technique introduced in Section 2, but for the present we will proceed loosely. First assume that A is purely discontinuous with "very small" jumps. Let ΔA_t denote the jump of A at time t (and bear in mind in the following caluclations that $\Delta A_t \leq 0$). We then have from (1.1) the approximation for "small" Δt

$$(1.3)\quad V(A_t,R_t) \cong \max_{c_t,\Delta A_t} \{u(c_t)\Delta t + P\Delta A_t + e^{-\rho(t+\Delta t)}E[V(A_t + \Delta A_t, R + \Delta R_t)]\}$$

$$\cong \max_{c_t,\Delta A_t} \{u(c_t)\Delta t + P\Delta A_t + (1 - \rho\Delta t)V(A_t,R_t) - V_R c_t \Delta t + V_A \Delta A_t - \Delta V(A,R)\Delta A_t\} ,$$

where

$$V_A(A,R) \equiv \frac{\partial V(A,R)}{\partial A} , \quad V_R \equiv \frac{\partial V(A,R)}{\partial R} , \quad \text{and}$$

$$\Delta V(A,R) \equiv \lim_{\Delta A \uparrow 0} E\left[\frac{V(A,\ R - \Delta R) - V(A,R)}{-\Delta A}\right] ,$$

that is, ΔV is the "expected rate" of improvement in V induced by additions to stockpiles R resulting from exploration at a unit rate (holding A constant). For example, under the Poisson distributional assumption, $\Delta V(A,R) = \beta[V(A,\ R + 1) - V(A,R)]$, where β is the parameter of the Poisson distribution.

Simplifying, we have the Bellman equation:

$$\rho V\Delta t \cong \operatorname*{Max}_{c_t \Delta A_t} \{u(c_t)\Delta t - V_R c_t \Delta t + [-\Delta V + P + V_A]\Delta A_t\} \quad . \tag{1.4}$$

From the first order conditions for optimality in (1.4) we have

$$c^* = C(V_R) \equiv u'^{-1}(V_R) \quad , \tag{1.5}$$

where u'^{-1} is the functional inverse of u', well defined since u is strictly concave. Furthermore,

$$\Delta A^* = 0 \quad \text{if} \quad \Delta V - P - V_A < 0 \quad , \tag{1.6}$$

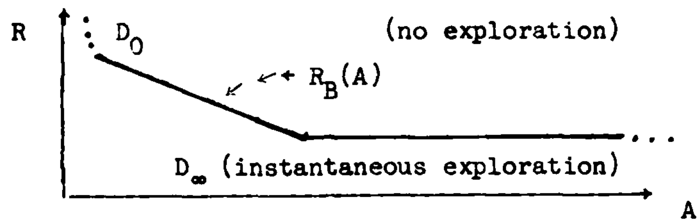
and otherwise:

$$\Delta A^* = \sup \{\Delta A < 0: \ \Delta V(A + \Delta A, R + \Delta R) - P - V_A(A + \Delta A, R + \Delta R) < 0 \ ; \ \underline{\underline{\text{or}}} \ A_t + \Delta A = 0\} \quad . \tag{1.7}$$

The recipe (1.6)-(1.7) for ΔA^* obviously needs some interpretation. The exploration policy A is apparently "bang-bang." If the marginal expected return to exploring, $\Delta V - P - V_A$, is negative, zero exploration is optimal (1.6). Otherwise, land should be explored instantaneously "until" either the random amount of resources extracted, ΔR in (1.7), is sufficient to push the marginal return on exploration below zero, or until the remaining land is exhausted. Note that ΔA_t^* is thus a random "probing variable." Instantaneous control for this problem is not merely a case of "infinite exploration rate," but also involves sequential decisions and realizations of random discoveries all at a single point of time! (See Section 2.)

Assuming all of the above can be made rigorous, the state space $\{(A,R) \in \mathbb{R}_+^2\}$ can be divided into two regions, no exploration and instantaneous exploration, as illustrated in Figure 1.1.

Figure 1.1 Control Regions in the State Space



In region D_0 (no exploration) we can eliminate the last term of the Bellman equation (1.4) since $\Delta A = 0$ and divide through by Δt, obtaining the following differential equation for V after substituting from (1.5):

$$\rho V(A,R) = u[C(V_R)] - V_R C(V_R) \quad , \quad (A,R) \in D_0 \quad . \tag{1.8}$$

Equation (1.8) is actually a first order differential equation for V as a function of R, of the form:

$$V(A,R) = W(R + R_E(A)) \quad , \tag{1.9}$$

where $R_E(A)$ is a constant representing the "resource equivalent" of the remainng unexplored land A. For the case $u(c) = \ln(c)$, $W(\cdot)$ has the solution

$$W(x) = \frac{1}{\rho} [\ln(\rho x) - 1] \quad , \tag{1.10}$$

which implies, using (1.5),

$$C^{*}\,(A,R) = \rho[R + R_E(A)] \quad . \tag{1.11}$$

Similar solutions are available for the case of power utilities and, in general,

$$c^{*}\,(A,R) = \frac{-\rho V_R}{V_{RR}} \quad , \tag{1.12}$$

which is the classical Hotelling [1931] result in the deterministic case ($A = R_E(A) = 0$).

Turning to the instantaneous exploration region D_∞, we can divide the Bellman equation (1.4) through by ΔA, eliminating terms in Δt since "instantaneous" of course means $\Delta t/\Delta A = 0$. This leaves the inequality, using (1.6)-(1.7),

$$\Delta V - P - V_A > 0 \qquad (A,R) \in D_\infty \quad . \tag{1.13}$$

Derzko and Sethi [1981] give a common sense argument that (1.13) must in fact hold with equality in D_∞, while H-C-K give a more formal argument based on the transition probability governing movement in D_∞.

Thus the equation

$$\Delta V(A,R) - P - V_A(A,R) = 0 \qquad (A,R) \in D_\infty \tag{1.14}$$

along with the (1.8) and the obvious boundary conditions constitute the information available to determine a solution for $V(A,R)$. The key elements of the solution are the two functions: $R_B(A)$, the boundary curve separating D_0 from D_∞; and $R_E(A)$, the "resource equivalents" function.

The papers by H-C-K and Derzko and Sethi [1981] both contain numerical approximations for $R_B(\cdot)$. Their results are similar although obtained by quite different routes. H-C-K also produced numerical approximations for $R_E(\cdot)$ and extended the price dynamics analysis of Arrow and Chang [1978].

Sundaresan [1983] includes a survey of research in optimal extraction of nonrenewable resources under uncertainty.

2. The Control Clock

Before proceeding to a diffusion formulation of Arrow's model, it is important to place the basic model on a rigorous foundation by formalizing the notion of instantantaneous control suggested in Section 1.

It is inherent in Arrow's model that, during exploration, decisions of whether or not to continue exploring may be made successively at a single point of time. This is a much stronger interpretation of "instantaneous control" than has usually been considered (e.g. Harrison and Taksar [1982]). Here one may explore a certain area of land, reevaluate the situation based on new discovery information, and perhaps continue exploring, all at the same point of time. It is implicit that there are two "clocks" running, one describing the passage of "real" time, the other tracking the running down of unexplored land. During instantaneous exploration the first clock is not running, while the second records the using up of unexplored land and generates information about the increm`ts of the discoveries proceeds, $N^A$. For a well defined control prob`, however, the control policy should be formally

adapted to some information structure[1] $F = \{F_\tau, \tau \geq 0\}$ where, roughly speaking, F_τ describes the information known at "time τ" on some "control clock." A convenient way to set up the control clock is to let "control time" be the sum of real time and cumulative land explored. Accordingly, we relate $t(\tau)$, (real time at τ on the control clock), and $L(\tau) \equiv A_0 - A(\tau)$ (land explored) by:

$$L(\tau) = \int_0^\tau \lambda_s ds \geq 0 \tag{2.1}$$

$$t(\tau) = \int_0^\tau (1 - \lambda_s) ds \geq 0 \tag{2.2}$$

so that

$$\tau = L(\tau) + t(\tau) \geq 0 \quad , \tag{2.3}$$

where $\lambda_\tau \in [0,1]$ is the "exploration intensity" (a control process). When $\lambda_\tau = 1$, real time is standing still (2.2) while exploration takes place at full intensity (instantaneously in real time). When $\lambda_\tau = 0$ no exploration is occurring. Finally $\lambda_\tau \in [0,1]$ implies that real time and exploration are running simultaneously.

With this control clock, $\lambda(\tau)$, $A(\tau)$, $t(\tau)$, $R(\tau)$, and $c(t(\tau))$ can all be formally modeled as stochastic processes adapted to some exogenous information structure. If the notion that exploration can actually occur instantaneously is rejected, the exploration control λ can be restricted to a compact subinterval of $[0,1]$.

It is a difficult task indeed to formalize the analysis in Section 1 (and in the earlier papers) without a device similar to this time

change. For example, it was assumed throughout the derivation of the Bellman equation that $\Delta\Delta_t$ is "very small," whereas the actual policy involves considerable jumps in certain regions of the state space.

3. A Diffusion Formulation of Arrow's Model

In this section we reformulate Arrow's model using a diffusion idealization of the discoveries process N^A. Although this does not improve on the Poisson distributional assumption by yielding an analytical solution (as one might have hoped), it does promote a somewhat clearer understanding of certain points. It also establishes the robustness of certain of Arrow and Chang's [1978] original results under quite different probabilistic assumptions.

As a preliminary, let $B = (B_\tau: \tau \geq 0; \Omega, F, P)$ denote a Standard Brownian Motion on the probability space (Ω, F, P).

We begin by suggesting that the exploration activity requires stochastic inputs $I = (I_\tau; \tau \geq 0)$ of the resource itself, where I_τ is the amount of the resource required to explore the first L_τ units of land area. The stochastic process $0 = (0_\tau, \tau \geq 0)$ similarly describes gross discoveries (output) of the resource.

If the input and output processes I and 0 have i.i.d. increments over each successive section of land of fixed area, then a natural idealization of the net discoveries process $N \equiv 0 - I$ is the controlled diffusion process:

$$N_\tau^\lambda = \int_0^\tau \lambda_s [\mu ds + \sigma dB_s] \tag{3.1}$$

for some positive constants μ and σ. See Harrison [1981] for the arguments leading to this idealization of a production process.

From (3.1) we see that N^λ has drift $\lambda_\tau \mu$ and "instantaneous variance" $\lambda_\tau^2 \sigma^2$ which depend on the exploration control λ. In effect, λ "speeds up" or "slows down" the (μ,σ) Brownian motion discoveries process which would result if exploration were carried out at a uniform rate.

The control problem (1.1)-(1.2)-(1.3) is unchanged, but can now be set up in "control time," using (2.1) and (2.2), as:

Find $V: \mathbb{R}^2 \to \mathbb{R}$ and non-anticipating controls λ_τ and $c_\tau \equiv c(t(\tau))$ satisfying

$$(3.2) \qquad V(A,R) = \underset{\lambda c}{\text{Max}}\ E_{AR}^{\lambda c}\left\{\int_0^\infty e^{-\rho t(\tau)}[u(c_\tau)(1 - \lambda_\tau) - P\lambda_\tau]d\tau\right\}$$

where

$$(3.3) \qquad dt(\tau) = (1 - \lambda_\tau)d\tau \quad , \quad t(0) = 0 \quad , \quad \lambda_\tau \in [0,1]$$

$$(3.4) \qquad dR_\tau = dN_\tau^\lambda - c_\tau(1 - \lambda_\tau)d\tau \quad , \quad R_\tau > 0$$

$$(3.5) \qquad dN_\tau^\lambda = \lambda_\tau \mu d\tau + \lambda_\tau \sigma dB_\tau \quad ,$$

$$(3.6) \qquad dA_\tau = -\lambda_\tau d\tau \quad , \quad A_\tau > 0 \quad .$$

To guarantee that the above integrals are well-defined (with V permitted to take the value $-\infty$), we assume that $c \in [0,\bar{c}]$ for some (large) constant $\bar{c}$. The conditions on u given in Section 1 then place an upper bound on $u(c_\tau)$.

We will proceed on the basis that a solution to (3.2) exists with

$$V_R \quad , \quad V_A \quad , \text{ and } \quad V_{RR} \equiv \frac{\partial^2 V}{\partial R^2}(A,R)$$

existing and bounded (at least for $A > 0$, $R > 0$). Using Ito's lemma, the infinitesimal generator $L^{\lambda c}$ corresponding to the transition semigroup for the controlled process N^λ yields (using (3.4), (3.5) and (3.6)):

$$\text{(3.7)} \qquad L^{\lambda c}V(A,R) = -\lambda V_A + V_R[\mu\lambda - c(1-\lambda)] + \lambda^2 \frac{\sigma^2}{2} V_{RR} \quad .$$

Then applying the strong Markov property of B and relation (3.3), we derive the Bellman equation from (3.2):

$$\text{(3.8)} \qquad V = \underset{\lambda c}{\mathrm{Max}} \left\{[1 - \rho(1-\lambda)]V + u(c)(1-\lambda) - P\lambda + L^{\lambda c}V\right\}$$

$$\text{subject to: } \quad c > 0 \quad , \quad \lambda \in [0,1] \quad .$$

(See, for example, Fleming and Rishel [1975] Chapter VI for details.) Note that with the advantages of a "control clock" and the continuity of diffusion processes, the derivation of the Bellman equation is quick and without approximation.

Since $u(\cdot)$ is strictly concave it is easy to deduce from (3.2) that $V_{RR} < 0$. We can then define

$$\text{(3.9)} \qquad \hat{\lambda}(A,R) \equiv \frac{-1}{V_{RR}\sigma^2}\left[\rho V - u(c(V_R)) - P - V_A + V_R(\mu + c(V_R))\right] \quad .$$

The first order necessary conditions for optimality in (3.8) then generate the following solutiuon for $\lambda^*(A,R)$ and $c^*(A,R)$, the optimal exploration and consumption controls:

$$\hat{\lambda}(A,R) \geq 1 \Rightarrow \lambda^*(A,R) = 1;\ c^*(A,R) \text{ arbitrary}$$

$$\text{(3.10)} \qquad 0 < \hat{\lambda}(A,R) < 1 \Rightarrow \lambda^*(A,R) = \hat{\lambda}(A,R);\ c^*(A,R) = C(V_R)$$

$$\hat{\lambda}(A,R) \leq 0 \Rightarrow \lambda^*(A,R) = 0;\ c^*(A,R) = C(V_R) \quad .$$

Combining (3.7), (3.8), and (3.10) and simplifying, we have the final form of the Bellman equation:

$$\text{(3.11)} \qquad (1 - \lambda^*)[-c^*V_R - \rho V + u(c^*)] + \lambda^*(\mu V_R - V_A - P) + \lambda^{*2} \frac{\sigma^2}{2} V_{RR} = 0 \quad .$$

Note that (3.11) is a partial differential equation for V which must hold throughout the state space. Just as in Section 1, however, we can simplify the Bellman equation by dividing up the state-space, this time into the <u>three</u> regions:

$$D_0 = \{(A,R) \in \mathbb{R}^2_+ : \hat{\lambda}(A,R) \leq 0\} \quad \text{(consumption only)}$$

$$D_1 = \{(A,R) \in \mathbb{R}^2_+ : \hat{\lambda}(A,R) \in (0,1)\} \quad \text{(simultaneous consumption and exploration)}$$

$$D_\infty = \{(A,R) \in \mathbb{R}^2_+ : \hat{\lambda}(A,R) \geq 1\} \quad \text{(instantaneous exploration)} \quad .$$

The intermediate region D_1, where exploration and consumption occur simultaneously and continuously, is also feasible for the formulation in Section 1, but can be rejected there as sub-optimal from the pure discontinuity of N^A. In the diffusion case, however, we see from

(3.9) that D_i is null only if there is a solution $\hat{V}(A,R)$ for the Bellman equation (3.11) satisfying:

$$(3.12) \qquad \rho\hat{V} = u\big((\hat{V}_R)\big) - P - \hat{V}_A + \hat{V}_R\big(\mu + C(\hat{V}_R)\big) = 0$$

with

$$\lambda^*(A,R) = \hat{\lambda}(A,R) = 0$$

and

$$c^*(A,R) = C(\hat{V}_R) \quad .$$

In the case of logarithmic utility, $u(c) = \ln(c)$, the following specification for $\hat{V}$ satisfies (3.11) and, if $P = 0$, (3.12):

$$\hat{V}(A,R) = \frac{1}{\rho}\ln\big(\rho(R + \mu A)\big) + \frac{P - 1}{\rho} \quad .$$

We feel this to be somewhat of a quirk, however, related to the very special properties of logarithmic utility $\big(C(V_R)V_R = 1\big)$. In general the solution is enriched by situations in which it is optimal to consume and explore simultaneously.

In D_0, the "consume only" region, the same ODE (1.9) found in Section 1 is obtained from the Bellman equation (3.11) for $\lambda^* = 0$. That is, it is again optimal to consume in D_0 according to the "Hotelling rule" (1.12) until the reduction in stocks induces further exploration.

Finally in D_∞, where $\lambda^* = 1$, we obtain from (3.11)

$$\mu V_R + \frac{\sigma^2}{2} V_{RR} - V_A - P = 0 \quad . \tag{3.14}$$

Note that (3.14) is the direct analogue of (1.14), which was obtained by a much more tortuous route.

In fact (3.14) and (1.14) can be shown to be formally equivalent by applying Ito's lemma to the definition of ΔV in (1.3).

In the diffusion case (3.14) has an interesting interpretation as the Kolmogorov Backward equation governing the transition density of V inside D_∞. Define the stopping time

$$T = \inf \{\tau > 0: (A_\tau, R_\tau) \notin D_\infty\}$$

and consider the stopped process $V(A_{\tau\wedge T}, R_{\tau\wedge T})$. Since $\lambda \equiv 1$ in D_∞, R moves in D_∞ simply as a (μ,σ^2) Brownian motion, and $dA_\tau \equiv d\tau$ from (2.1). For any stopping time $s < T$, by Bellman's Principle of Optimality we have, for $(A,R) \in D_\infty$

$$\begin{aligned} V(A,R) &= E[V(A_s,R_s) - P(A_s - A_\tau) | A_\tau = A, R_\tau = R] \\ &= E[V(A - s + \tau, R_s) - P(s - \tau) | R_\tau = R] \quad . \end{aligned} \tag{3.15}$$

In particular, for $s = T$, (3.15) says that the current value of V at a point in D_∞ is the expected value of V at the exit point (A_T, R_T), less the expected exploration cost to "get out," $E[P(T - \tau)]$.

Now define $Y(s,\tau,R)$ by equating it with the right-hand side of (3.15). Then (3.14) is formally the backward equation for $Y(s,\tau,R)$, since R_τ is a (μ,σ^2) Brownian motion in D_∞. Thus, since

$Y(\tau,\tau,R) = V(A,R)$ for $(A,R) \in D_\infty$, (3.14) is the partial differential equation giving the transition density of V, treating land as "time."

4. Mapping from Control Time Back to Real Time

In the last section we characterized the optimal controls λ^* and c^* as feedback policies on the current state variables, unexplored land (A) and stockpiles of the resource (R). Given the simple relationship (2.1) between "real time" and "control time," it is simple to map these policies, which apply in control time, back into real time.

Since the consumption policy $c_\tau \equiv c(t(\tau))$ was formulated in real time to begin with, we have the solution

$$c^*(t) = C(V_R(A_t,R_t)) \quad , \tag{4.1}$$

which holds everywhere (since the consumption policy is irrelevant in D_∞).

The exploration policy must be characterized separately in each of the regions from (3.6) and (3.10). In D_∞, it is impossible to express the policy in real time except to say that it is optimal to explore "instantaneously" to the boundary of D_∞. In some of the previously cited studies it has been overlooked that this may entail exploration of all remaining land without ever reaching the boundary curve $R_B(\cdot)$ illustrated in Figure 1. In the case of diffusion discoveries, it may even occur that instantaneous exploration will exhaust all stocks R before the remaining land is all explored. Of course the probability of such an occurrence will be small given reasonable choices for μ and σ.

In D_0, by definition, the optimal exploration policy A^* is

(4.2) $$\dot{A}^*(t) \equiv \frac{dA^*(t)}{dt} = 0 \quad .$$

Finally, in the intermediate region we have

(4.3) $$\dot{A}^*(t) = \frac{-\hat{\lambda}(A_t, R_t)}{1 - \hat{\lambda}(A_t, R_t)} \quad ,$$

where $\hat{\lambda}$ is the function defined in (3.9).

Footnotes

1/ Formally, we should preface these remarks by a statement that (Ω,F,P) is a probability space describing uncertainty in Arrow's model. Then $\mathbb{F}$ is a filtration, a right-continuous increasing family of sub-tribes of F, with $F = \bigcup_{\tau>0} F_\tau$. See, for example Meyer [1967], p. 65.

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